HOMEWORK SET 0: REMEMBERING MATH
Due: Monday, August 28, 2023

To get your math neurons firing again, perform the following operations (YOU MAY USE THE CRC):

$$\frac{d}{dt}e^{-kt} = \qquad \frac{d}{dt}\frac{1}{kt} = \qquad \frac{d}{dt}\ln(kt) =$$

$$\int e^{-kt}dt = \qquad \int \frac{dt}{kt} = \qquad \int \ln(t)dt =$$

$$\int \frac{dt}{1+kt} =$$

$$\int \frac{tdt}{1+kt} =$$

$$\int \frac{tdt}{1+kt^2} =$$

The hyperbolic functions that we'll use are defined as

$$\sinh(z) = \frac{1}{2}\left(e^{z} - e^{-z}\right) \qquad \cosh(z) = \frac{1}{2}\left(e^{z} + e^{-z}\right)$$

Use the first one to show that (*"show" means start with the first expression and derive the second*)

$$\sinh^{-1}(z) = w \quad \Rightarrow \quad \sinh^{-1}(z) = \ln\left(z + \sqrt{z^2+1}\right)$$

(hint: start with $2z = e^{w} - e^{-w}$ and solve for w … you'll have to solve a quadratic in e^{2w} (& e^{w} & 1) and note that since the radical is always ±, technically, ±√ = +√)

Then show that

$$\frac{d}{dz}\ln\left(z + \sqrt{z^2+1}\right) = \frac{1}{\sqrt{z^2+1}}$$

Then show that (THE CRC WILL GIVE YOU THE FIRST EXPRESSION. SHOW HOW TO GET THE SECOND.)

$$\frac{d}{dz}\cosh^{-1}\left(e^{kz}\right) = \frac{k}{\sqrt{1-e^{-2kz}}} = \frac{ke^{kz}}{\sqrt{e^{2kz}-1}}$$